Ratu Navula College

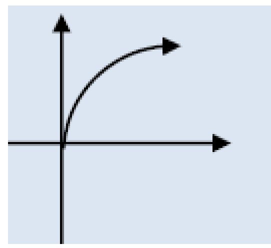
Year 12 Mathematics Lessons Notes – Week 1

<u>Strand 3: Graphs</u> **<u>Sub Strand: 3.1 Graphs and Intersections</u>**

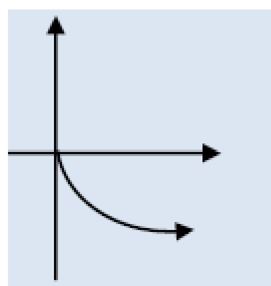
<u>Lesson 38: Square Root $\sqrt{x}$ Graphs</u>

<u>Learning Outcome</u>: Sketch the square graph by using transformation method.

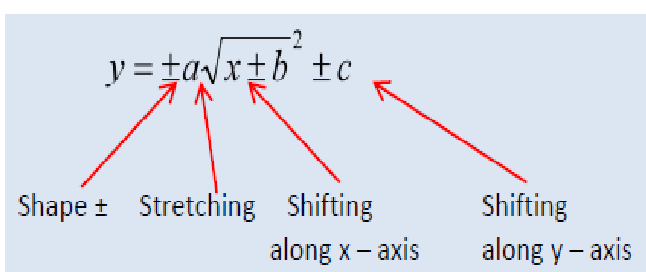
- For $y = +\sqrt{x}$ the graph will be:



- For $y = -\sqrt{x}$ the graph will be:

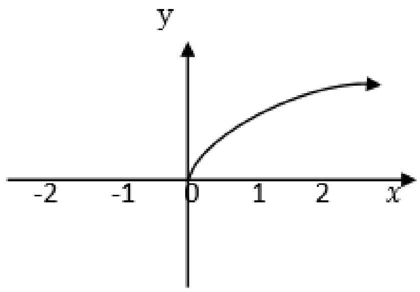


The transformation for a square root graph will be of the form:



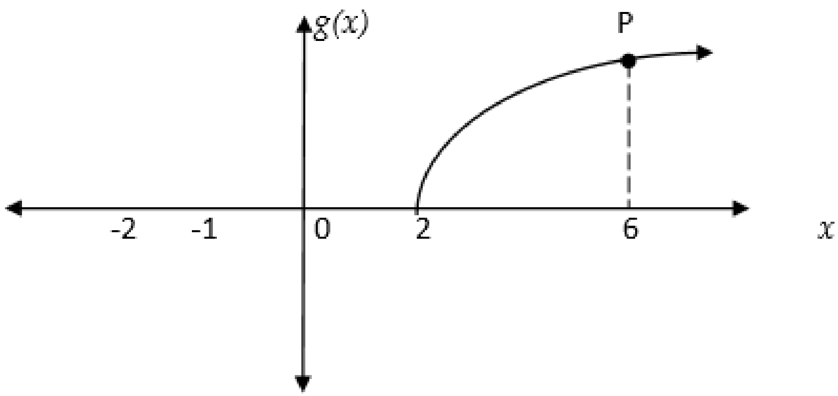
EXAMPLE 1: Draw the graph of $y = \sqrt{x}$ and State the domain

x	- 1	0	1	2	4
$y = \sqrt{x}$	$\sqrt{-1} = undefined$	$\sqrt{0} = 0$	$\sqrt{1} = 1$	$\sqrt{2} = 1.41$	$\sqrt{4} = 2$



Domain = {x: x ≥ 0, $x \in R$}

EXAMPLE 2: The diagram below shows the graph of $g(x) = \sqrt{x-2}$



a) Write down the coordinates of point P.
b) Find g (- x).
c) On the pair of axis drawn, draw the graph of g (- x).
d) Describe the transformation.
e) State the domain and range of g (- x).

Answers:

a) coordinates of point P
x = 6, Substitute

$g(6) = \sqrt{6-2}$
$= \sqrt{4} = 2 \therefore (6,2)$

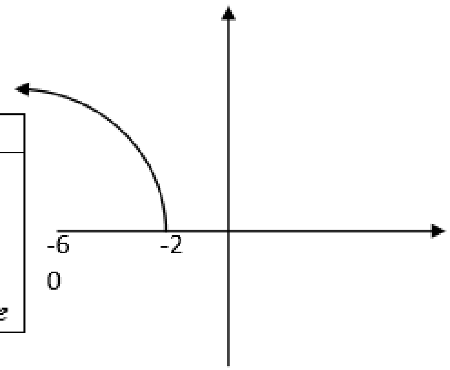
b) Substitute $-x$ in place of x

$g(x) = \sqrt{x-2}$

$g(-x) = \sqrt{-x-2}$

c) Use tables:

x	-2	-1	0
$g(-x) = \sqrt{-x-2}$	$g(--2)$ $= \sqrt{--2-2}$ $= 0$	$g(--1)$ $= \sqrt{--1-2}$ $= \sqrt{-1}$ $=$ ***undefine***	$g(-0)$ $= \sqrt{-0-2}$ $= \sqrt{-2}$ $=$ ***undefine***

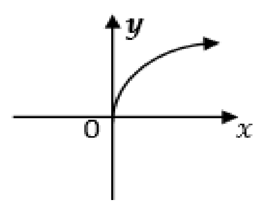


d) Reflection in the y - axis

e) Domain $\{x : x \leq -2, x \in R\}$
Range $\{y : y \geq 0, y \in R\}$

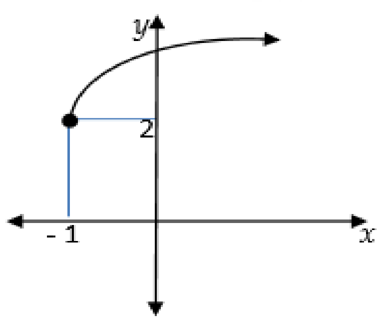
Class Activity 38

1. State domain and range of the graph:



2. Sketch the graph of $y = -\sqrt{x-2}$ and state the domain and range.

3. The graph of the function $g(x)$ is shown below.



a) Write the equation of $g(x)$.
b) Write down the y – intercept of $g(x)$
c) State the domain and range of $g(x)$

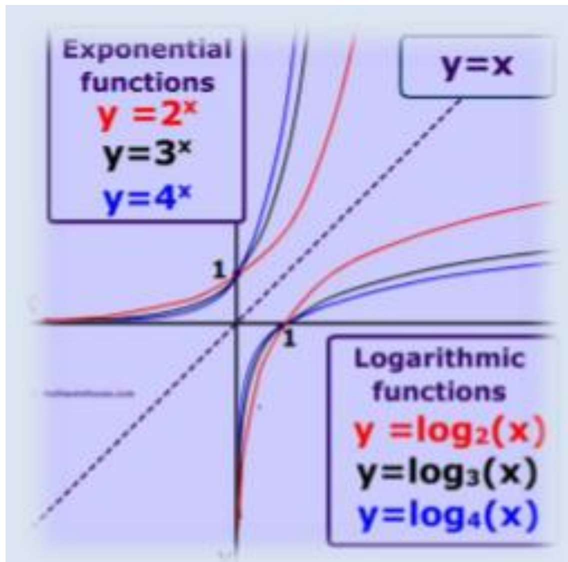
Strand 3: Graphs **Sub Strand: 3.1 Graphs and Intersections**

Lesson 39: Logarithmic/Exponential Graphs

Learning Outcome: Sketch the logarithmic and exponential graph using table method.

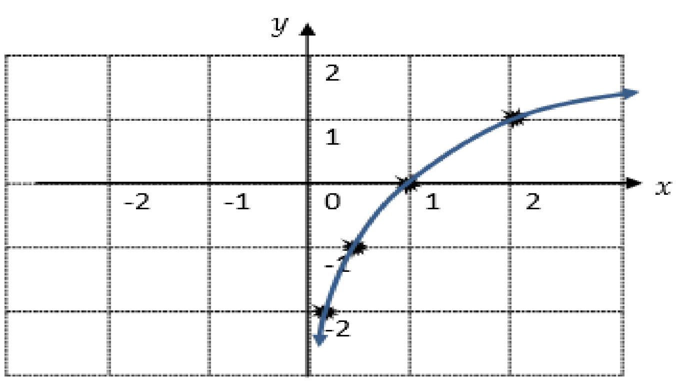
Note:

Exponential and Logarithmic graphs are inverses or mirror reflections of each other



EXAMPLE 1: Sketch graph of $y = \log_2 x$

x	0.125	0.25	0.5	1	2
$y = \log_2 x = \frac{\log x}{\log 2}$	-3	-2	-1	0	1



- **The y-axis is an asymptote because log 0 is undefined**

EXAMPLE 2: A function is given as $f(x) = 3^x$

i. Find the coordinates of the y – intercept
ii. Sketch the graph of *f(x) and* label it clearly

Another function is defined as $g(x) = \log_3 x$

iii. On the pair of axes, Sketch the graph of g*(x),* showing the x – intercept clearly

iv. Describe fully the transformation that maps the graph of f*(x)* onto the graph of g*(x).*

Answers:

i. y – int, let x = 0 and solve

$f(x) = 3^x$

$\therefore y = 3^x$

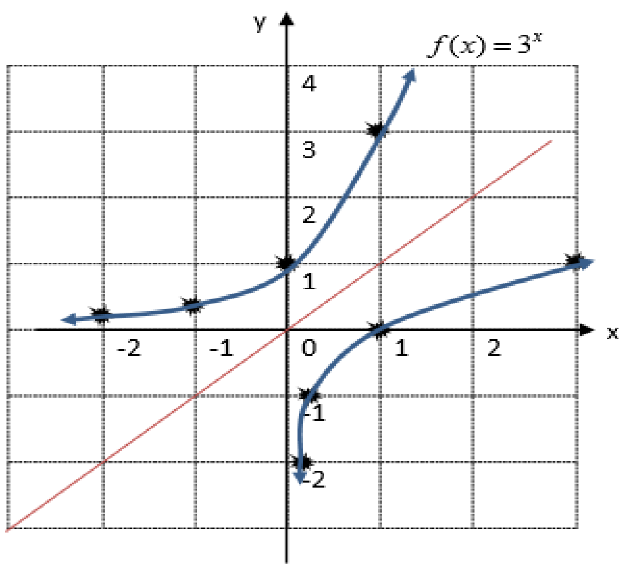
$= 3^0 = 1$

$(0,1)$

ii. Sketch: using tables

x	-2	-1	0	1	2
$y = 3^x$	$y = 3^{-2} = \frac{1}{9}$	$y = 3^{-1} = \frac{1}{3}$	$y = 3^0 = 1$	$y = 3^1 = 3$	$y = 3^2 = 9$

ii / iii Sketch



Prove that $f(x) = 3^x$ and $y = \log_3 x$ are inverses of each other.
To find the inverse:

$y = 3^x$

Interchange **x and y**

$3^y = x$
$\log 3^y = \log x$
$y = \dfrac{\log x}{\log 3}$
$y = \log_3 x$

iv. Reflection in the line $y = x$.

<u>Class Activity 39</u>

1. Sketch the following graphs:

   a) $y = \log x$ b) $y = \log_3 x$

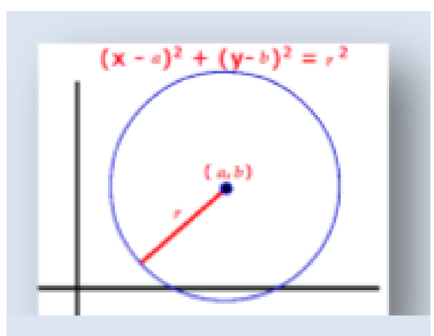
2. Sketch the following graphs:
   a) $y = -2^x$
   b) $y = 4^{-2x}$
   c) $f(x) = -0.5^x$

<u>Strand 3: Graphs</u> **<u>Sub Strand: 3.1 Graphs and Intersections</u>**

<u>Lesson 40: Graphs of Circles:</u>

<u>Learning Outcome</u>: Determine the radius and sketch the graph of the circle

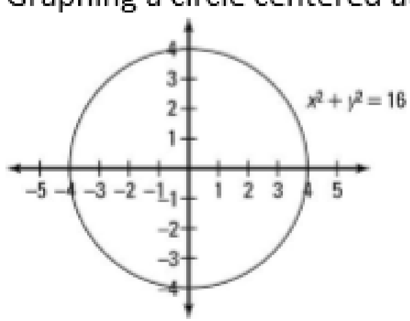
- The equation of a circle at the centre of the origin (0,0) with the radius r is given as $x^2 + y^2 = r^2$
- The general equation of a circle with the radius r and the centre (a,b) is $(\boldsymbol{x} - \boldsymbol{a})^2 + (\boldsymbol{y} - \boldsymbol{b})^2 = \boldsymbol{r}^2$



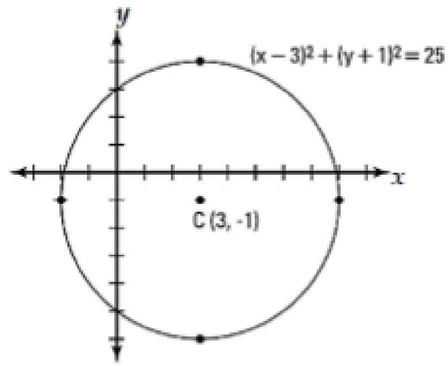
<u>Example</u>

1. Sketch the graph $x^2 + y^2 = 16$

Graphing a circle centered at the origin with $r^2 = 16$, thus r = 4.



2. Sketch the equation of $(x-3)^2 + (y+1)^2 = 25$



Centre of (3, -1) with a radius of 5.

Class activity 40

Sketch the following graphs

a) $x^2 + y^2 = 3^2$

b) $x^2 + y^2 = 36$

c) $(x-2)^2 + (y+3)^2 = 9$

Strand 3: Graphs **Sub Strand 3.2 : Simultaneous Equation**

Lesson 41: Application of Simultaneous Equation

Learning Outcome: Use elimination, substitution method to solve the simultaneous equations.

- Simultaneous means solve for variables at the same time.
- Three methods includes are:
  - ✓ Elimination method
  - ✓ Substitution method
  - ✓ Graphical method

❖ **Elimination method**

- Inorder to eliminate one variable, either add or subtract the two equation.
- Solve for one variable first
- Substitute in any of the equation to get the next variable.

❖ **Substitution method**

- It means put one equation into another equation

Example 1

Solve these two equations simultaneously.

$4x - 3y = 3$ *and* $10x + 3y = 4$

$$\begin{array}{rl} & 4x - 3y = 3 \\ + & \underline{10x + 3y = 4} \\ & 14x = 7 \\ & x = \tfrac{1}{2} \text{ or } 0.5 \end{array}$$

Substitute in any equation:

$4x - 3y = 3$

$4(0.5) - 3y = 3$

$2 - 3y = 3$

$2 - 2 - 3y = 3 - 2$

$-3y = 1$

$y = -\frac{1}{3}$

Example 2

To solve the above graphs we will need to make sure that y is the subject in both equations.

$3y = x + 1 \quad and \quad 2y - 4x - 4 = 0$

$3y = x + 1 \rightarrow y = \frac{x+1}{3}$

$2y - 4x - 4 = 0 \rightarrow \rightarrow 2y = 4x + 4 \rightarrow y = 2x + 2$

$y = \frac{x+1}{3} \quad y = 2x + 2$

$y = y$

$\frac{x+1}{3} = 2x + 2$, $x + 1 = 3(2x + 2)$ $\rightarrow$ $x + 1 = 6x + 6$, $x = 6x + 6 - 1$ $\rightarrow$ $x = 6x + 5$, $x - 6x = 5$ $\rightarrow$ $-5x = 5$, $x = -1$

$y = 2x + 2 \rightarrow y = 2(-1) + 2 \rightarrow y = -2 + 2 \rightarrow y = 0$

Point of intersection is (-1, 0)

Example 3

A total of 925 tickets were sold for $5925. If adult tickets cost $7.50 and children's tickets cost $3.00, how many tickets of each kind were sold?

Answers:
Let x be the number of adult tickets.
Let y be the number of children's tickets.

$x + y = 925$ Multiply this equation by 3 and subtract
$7.5x + 3y = 5925$

$$\begin{array}{rl} & 3x + 3y = 2775 \\ - & 7.5x + 3y = 5925 \\ \hline \end{array}$$

$-4.5x = -3150$
$x = 700$

Substitute in any equation:

$x + y = 925$
$700 - 700 + y = 925 - 700$
$Y = 225$

Thus 700 adult tickets and 225 children's tickets.

Class Activity 41

1. Two numbers have a sum of 90 and one is 5 times the other.

   a) Write down the pair of simultaneous equation.
   b) Solve for the values of the two numbers.

2. The total number of girls and boys in a class is 42. There are more girls than boys in the class. The difference between the number of boys and girls is 16. Find the number of girls and boys in the class.

Strand 3: Graphs **Sub Strand 3.2 : Simultaneous Equation**

Lesson 42: Linear and Quadratic Equation

Learning Outcome: Find the point of intersection between linear and quadratic graph.

Quadratic and linear graphs always meet at two places.

Example 1

Find the coordinates of point of intersection of the parabola $y = x^2 - 4$ and linear graph $y = x + 2$.

$y = x^2 - 4$ *and* $y = x - 2$

$y = y$

$x^2 - 4 = x - 2$

$x^2 - 4 - x + 2 = 0$

$x^2 - x - 4 + 2 = 0$

$x^2 - x - 2 = 0$

→ *factorization*

$x \rightarrow -2$

$x \rightarrow 1$

$-2x + 1x = -x \leftarrow$ *middle term*

$(x - 2)(x + 1) = 0$

$x - 2 = 0$	$x + 1 = 0$
$x = 0 + 2$	$x = 0 - 1$
$x = 2$	$x = -1$

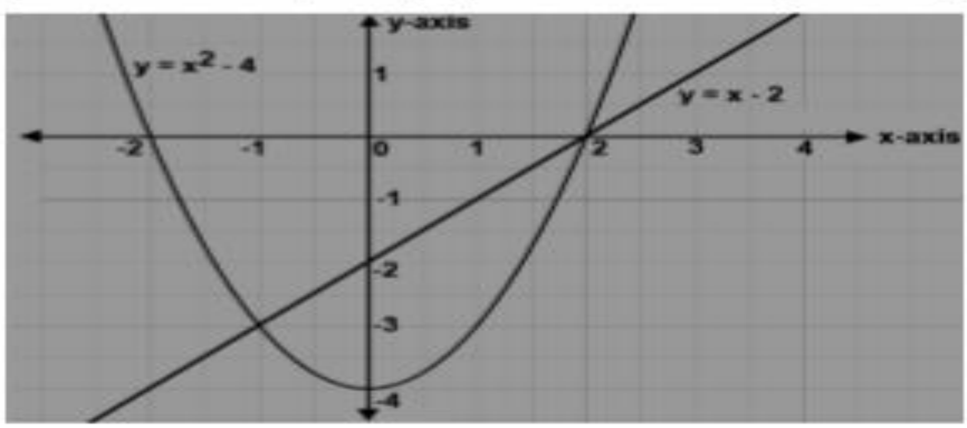
- Since there are two x values we can conclude that both graphs intersect at two different points.

$x = 2 \rightarrow y = x - 2$

$y = 2 - 2 = 0 \rightarrow (2, 0)$

$x = -1 \rightarrow y = x - 2$

$y = -1 - 2 = -3 \rightarrow (-1, -3)$



Example 2

Find the point of intersection of the graph $y = x^2 + 3x + 2$ *and* $y = x + 1$.

At the point of intersection, both graphs will have the same y-value. Hence, we can solve both equations simultaneously.

$$\begin{aligned} y &= y \\ x^2 + 3x + 2 &= x + 1 \\ x^2 + 3x + 2 - x - 1 &= 0 \\ x^2 + 3x - x + 2 - 1 &= 0 \\ x^2 + 2x + 1 &= 0 \end{aligned} \rightarrow$$

Now solve $x^2 + 2x + 1 = 0$

$x \rightarrow 1 \quad 1x$

$x \rightarrow 1 \quad 1x \quad 1x + 1x = \boxed{2x} \leftarrow$ |*middle term*|

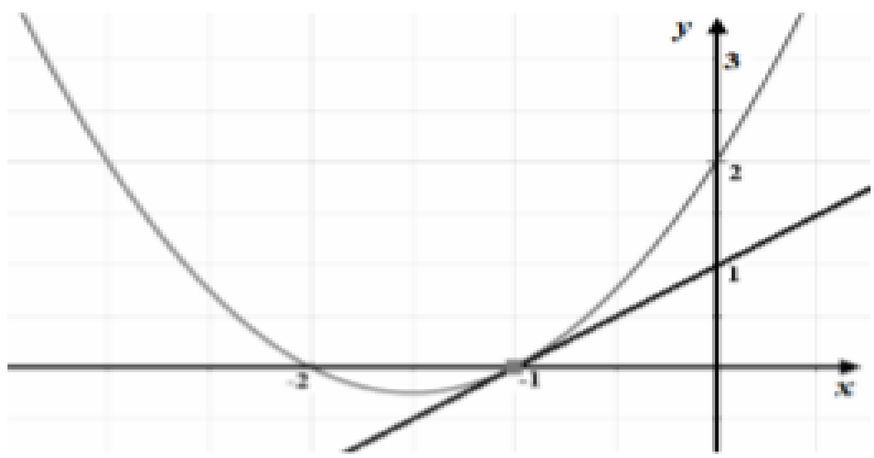
$x^2 + 2x + 1 = (x+1)(x+1) = 0$

$x + 1 = 0 \rightarrow x = 0 - 1 \rightarrow \boxed{x = -1}$

To find the y value, use any of the given equations.

$y = x + 1 \rightarrow y = -1 + 1 \rightarrow \boxed{y = 0}$ *Point of intersection* $\rightarrow \boxed{(-1,\ 0)}$

Given below is the graph of $y = x^2 + 3x + 2$ *and* $y = x + 1$. In the graph, the point of intersection is represented by a thick dot.



Class Activity 42

1. Determine the coordinates of the point where the parabola $y = x^2 - 6x + 8$ meets with the line $y = -2x + 4$.
2. Find the point of intersection of the line $y = 3x + 1$ with the parabola $y = x^2 - 3$

Strand 3: Graphs **Sub Strand 3.2 : Simultaneous Equation**

Lesson 43: Linear and Hyperbola Equation

Learning Outcome: Find the point of intersection between linear and hyperbola graph.

Hyperbolic equation and linear graph will meet at two places.

Example 1

Find the point of intersection $y = \frac{1}{x}$ and $y = \frac{x}{4}$

$y = \frac{1}{x} \quad y = \frac{1}{4}x$

Solve both equations simultaneously

$y = y$

$\frac{1}{x} = \frac{1}{4}x \rightarrow \frac{1}{x} = \frac{x}{4} \rightarrow 1 \times 4 = x \times x \rightarrow 4 = x^2$

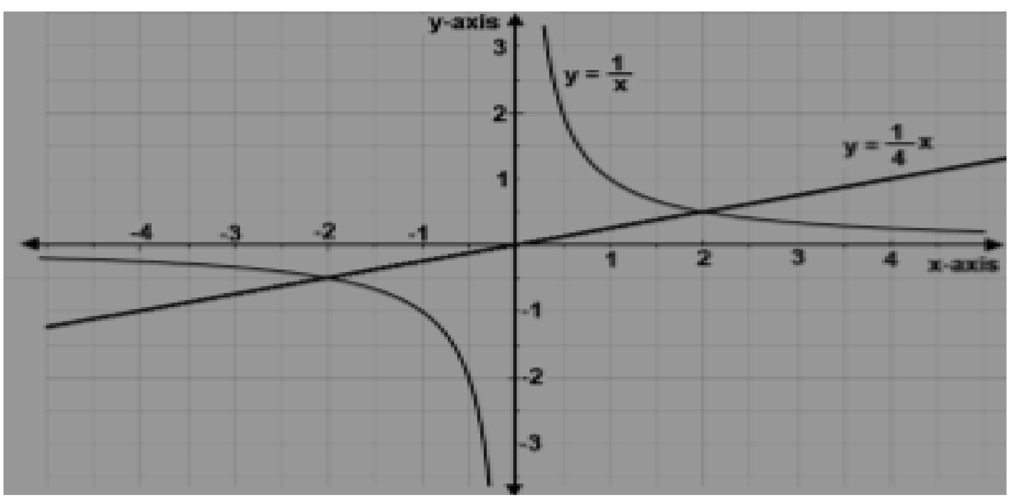
$4 = x^2 \rightarrow x^2 = 4 \rightarrow x = \pm\sqrt{4} \rightarrow x = \pm 2$

Substitude the x value into any of the equations

$x = 2$: $y = \frac{1}{x} \rightarrow y = \frac{1}{2}$

$x = -2$: $y = \frac{1}{x} \rightarrow y = -\frac{1}{2}$

points of intersection $\rightarrow \left(2, \frac{1}{2}\right)$ *and* $\left(-2, -\frac{1}{2}\right)$



Example 2

Find the point of intersection of $y = \frac{1}{x}$ and $y = x$

$y = \frac{1}{x} \quad y = x$

Solve both equations simultaneously

$y = y$, $\frac{1}{x} = x \rightarrow \frac{1}{x} = x \rightarrow 1 = x \times x \rightarrow 1 = x^2$

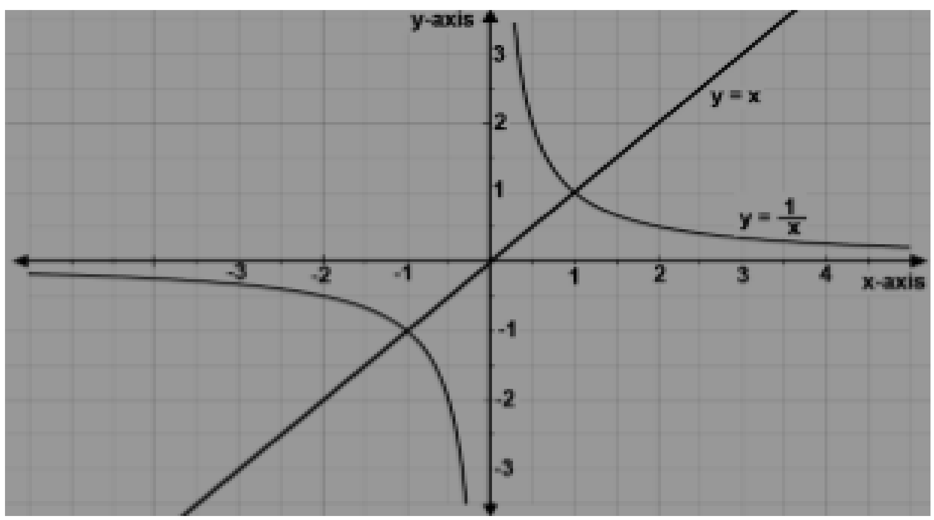
$1 = x^2 \rightarrow x^2 = 1 \rightarrow x = \pm\sqrt{1} \rightarrow x = \pm 1$

Substitude the x value into any of the equations

$x = 1$: $y = \frac{1}{1} \rightarrow y = 1$

$x = -1$: $y = -\frac{1}{1} \rightarrow y = -1$

points of intersection $\rightarrow (1, 1)$ *and* $(-1, -1)$



Class activity 43

1. Find the point of intersection of the curve $y = -\frac{2}{x}$ with the line $y = x - 3$
2. Find the point of intersection of $y = \frac{5}{x-2}$ and the straight line $y = 2x - 1$

Strand 3: Graphs   **Sub Strand 3.2 : Simultaneous Equation**

Lesson 44: Linear Equation and Circle

Learning Outcome: Use elimination, substitution method to solve the simultaneous equations

Circles and linear equation meets at two points

Example 1
Find the point of intersection $x^2 + y^2 = 25$ and $y = x$

$x^2 + y^2 = 5^2$
$y = x$

$\rightarrow$ *Substitute* $y = x$ *into* $x^2 + y^2 = 25$

$x^2 + (x)^2 = 25 \rightarrow 2x^2 = 25$

Now solve for x

$2x^2 = 25$
$x^2 = 25 \div 2$

$\rightarrow$ $x^2 = 12\cdot5$
$x = \mp\sqrt{12\cdot5}$

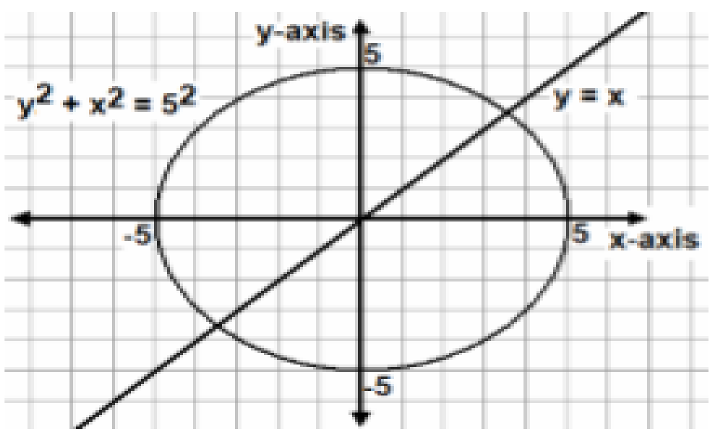
$\rightarrow$ $x = \pm3\cdot54$

Substitute the $x-$*value* in *the equation* $y = x$

$x = 3\cdot54$	$x = -3\cdot54$
$y = x$	$y = x$
$y = 3\cdot54$	$y = -3\cdot54$

Points of intersections

$(3\cdot54, 3\cdot54)$ *and* $(-3\cdot54, -3\cdot54)$



Class Activity 44

1. Find the point of intersection of the graphs $x^2 + y^2 = 2$ and $y + x - 2 = 0$
2. Find the point of intersection of the circle $x^2 + y^2 = 25$ and the straight line $y - x + 3 = 0$