# STRAND 2 MEASUREMENT

## SUSBSTRAND 2.1 UNITS OF MEASUREMENT AND PREFIXES

### LESSON 30

LO: Give the basic SI units.

: State the values of commonly used prefixes.

**SI units ( Standard Units) and Symbols**

- "International System."
- **Standard unit:** well defined units that the whole world agrees on.
- **Basic units:** metre, kilogram, second, ampere, Kelvin, candela, mole, radian.
- **Derived units:** all other units are made from basic units.

Table 5.1 Basic SI units

| Quantity | Unit | Symbol |
|---|---|---|
| Length | metre | m |
| Mass | kilogram | kg |
| Time | second | s |
| Electric current | ampere | A |
| Light | candela | cd |
| Temperature | Kelvin | K |
| Amount | mole | mol |
| Angle | radian | rad |

**ACTIVITY**

5.2 Units of Measurement

2. Which SI unit would you use to measure the following quantities? Write the name of the unit and its symbol.

(a) mass
(b) distance
(c) amount
(d) temperature
(e) light intensity
(f) electric current
(g) time
(h) angle

3. Write symbols for the following units:

(a) kilogram
(b) second
(c) ampere
(d) metre
(e) candela
(f) radian
(g) mole
(h) Kelvin

4. Write the names of the following units:

(a) A
(b) mol
(c) s
(d) rad
(e) kg
(f) K
(g) m
(h) cd

| Quantity | Unit | Symbol | Value |
|---|---|---|---|
| Length | metre | m | Formerly, 1 ten-millionth of the distance from the equator to the North Pole. Now defined as the distance travelled by light in a vacuum during $^1/_{299,792,458}$ of a second. |
| | centimetre | cm | 100 cm = 1 m |
| | millimetre | mm | 1000 mm = 1 m |
| | kilometre | km | 1 km = 1000 m |
| | nautical mile | n mile | 1 n mile = 1852 m |
| Area | square metre | $m^2$ | (1 m × 1 m) |
| | square centimetre | $cm^2$ | 10,000 $cm^2$ = 1 $m^2$ |
| | hectare | ha | 1 ha = 100 m × 100 m = 10,000 $m^2$ |
| Volume | cubic metre | $m^3$ | (1 m × 1 m × 1 m) |
| | cubic centimetre | $cm^3$ | 1,000,000 $cm^3$ = 1 $m^3$ |
| | litre | L | 1000 L = 1 $m^3$ |
| | millilitre | mL | 1 mL = 1 $cm^3$ |
| Mass | kilogram | kg | Formerly, the mass of 1 $dm^3$ of pure water at 4°C. Now defined as the mass of a platinum-iridium cylinder kept in Paris, France. |
| | gram | g | 1000 g = 1 kg |
| | milligram | mg | 1000 mg = 1 g |
| | tonne | t | 1 t = 1000 kg |
| Time | second | s | Formerly, $^1/_{86,400}$ of a day. Now defined in terms of the resonant frequency of a caesium atom: 9,192,631,770 cycles per second. |
| | minute | min | 1 min = 60 s |
| | hour | h | 1 h = 60 min |
| | day | d | 1 d = 24 h |
| Angle | degree | ° | One degree is defined as $^\pi/_{180}$ radians. |
| | minute | ' | 1' = $^1/_{60}$ of a degree |
| | second | " | 1" = $^1/_{60}$ of a minute |
| Temperature | degrees Kelvin | °K | 273.16°K is defined as the triple point of water, where all three phases (solid, liquid, and gas) are in equilibrium. |
| | degrees Celsius | °C | 0°C = 273.15°K (the freezing point of water) |
| Speed | metres per second | m/s | |
| | kilometre per hour | km/h | 1 m/s = 3.6 km/h |
| Concentration | grams per litre | g/L | |
| | moles per litre | mol/L | |
| Density | grams per cubic cm | $g/cm^3$ | |
| Data | bit | b | In the base 2 system of mathematics, a bit is one digit: either 0 or 1 |
| | byte | B | 1 byte = 8 bits |
| | kilobyte | kB | 1 kB = 1024 bytes ($2^{10}$ bytes)* |
| | megabyte | MB | 1 MB = 1024 kB ($2^{20}$ bytes)* |
| | gigabyte | GB | 1 GB = 1024 MB ($2^{30}$ bytes)* |

*Strictly speaking, 1 kB is equal to 1000 bytes. However, in computing, amounts of data are measured in the base 2

## SI Prefixes

- Used to change the size of our units.
- Multiply or divide a unit by power of 10.
  Example: kilo means "x 1000"
  milli means " ÷ 1000"

**Table 5.2 Common SI prefixes**

| Prefix | Symbol | Factor | Example |
|---|---|---|---|
| mega | M | $10^6$ (1,000,000) | 1 Mm = 1,000,000 m |
| kilo | k | $10^3$ (1000) | 1 km = 1000 m |
| hecto | h | $10^2$ (100) | 1 hm = 100 m |
| deca | da | $10^1$ (10) | 1 dam = 10 m |
| deci | d | $10^{-1}$ ($^1/_{10}$) | 1 dm = 0.1 m |
| centi | c | $10^{-2}$ ($^1/_{100}$) | 1 cm = 0.01 m |
| milli | m | $10^{-3}$ ($^1/_{1000}$) | 1 mm = 0.001 m |
| micro | μ | $10^{-6}$ ($^1/_{1,000,000}$) | 1 μm = 0.000001 m |

**Example 1** Find the number of **(a)** grams in 15 kilograms, and **(b)** milliamperes in 1 ampere.

**(a)** grams in 15 kg

$1\text{kg} = 1000\text{ g}$

$\therefore 15\text{kg} = 15 \times 1000\text{ g} = 15{,}000\text{ g}$

**(b)** Milliamperes in 1 A

$1\text{mA} = \frac{1}{1000}\text{ A}$

$\therefore 1000\text{ mA} = 1\text{A}$

**Explanation:** There are 1000 g in 1 kg, so there are 15,000 g in 15 kg **(a)**. One mA is $^1/_{1000}$ A, so there are 1000 mA in 1 A **(b)**.

**How many centimetres are in a metre?**

If 1 cm = $^1/_{100}$ m, then there are 100 cm in 1 m:

$1\text{ cm} = \frac{1}{100}\text{ m}$

$\therefore 100\text{ cm} = 1\text{ m}$

The same is true for millimetres and micrometres:

$1\text{ mm} = \frac{1}{1000}\text{ m}$

$\therefore 1000\text{ mm} = 1\text{ m}$

and

$1\text{ μm} = \frac{1}{1{,}000{,}000}\text{ m}$

$\therefore 1{,}000{,}000\text{ μm} = 1\text{ m}$

**SI Prefixes**

| Example | Prefix | | Multiply by | | Reverse Example |
|---|---|---|---|---|---|
| One terawatt equals one trillion ($10^{12}$) watts. | tera- | T | trillion | $10^{12}$ | One watt equals one trillionth ($10^{-12}$) terawatts. |
| One gigawatt equals one billion ($10^{9}$) watts. | giga- | G | billion | $10^{9}$ | One watt equals one billionth ($10^{-9}$) gigawatts. |
| One megawatt equals one million ($10^{6}$) watts. | mega- | M | million | $10^{6}$ | One watt equals one millionth ($10^{-6}$) megawatts. |
| One kilowatt equals one thousand ($10^{3}$) watts. | kilo- | k | thousand | $10^{3}$ | One watt equals one thousandth ($10^{-3}$) kilowatts. |
| | | | | | |
| One centimetre equals one hundredth ($10^{-2}$) of a metre.* | centi- | c | hundredth | $10^{-2}$ | One metre equals one hundred ($10^{2}$) centimetres. |
| One millimetre equals one thousandth ($10^{-3}$) of a metre. | milli- | m | thousandth | $10^{-3}$ | One metre equals one thousand ($10^{3}$) millimetres. |
| One micrometre equals one millionth ($10^{-6}$) of a metre. | micro- | μ | millionth | $10^{-6}$ | One metre equals one million ($10^{6}$) micrometres. |
| One nanometre equals one billionth ($10^{-9}$) of a metre. | nano- | n | billionth | $10^{-9}$ | One metre equals one billion ($10^{9}$) nanometres. |
| One picometre equals one trillionth ($10^{-12}$) of a metre. | pico- | p | trillionth | $10^{-12}$ | One metre equals one trillion ($10^{12}$) picometres. |

*One centimetre equals $10^{-2}$ metre, but it is easier to think of it in reverse: one metre equals $10^{2}$ centimetres.

**Indices of 10**

| Word | Decimal | Index |
|---|---|---|
| Trillion | 1,000,000,000,000 | $10^{12}$ |
| Billion | 1,000,000,000 | $10^{9}$ |
| Million | 1,000,000 | $10^{6}$ |
| Thousand | 1,000 | $10^{3}$ |
| Hundred | 100 | $10^{2}$ |
| | | |
| Hundredth | 0.01 | $10^{-2}$ |
| Thousandth | 0.001 | $10^{-3}$ |
| Millionth | 0.000001 | $10^{-6}$ |
| Billionth | 0.000000001 | $10^{-9}$ |
| Trillionth | 0.000000000001 | $10^{-12}$ |

**Use of Prefixes**

1. You may combine many prefixes with one unit:

kilo-<br>milli-<br>micro- → metre → kilometre / millimetre / micrometre

2. Or, one prefix with many units:

kilo- → metre → kilometre<br>kilo- → gram → kilogram<br>kilo- → watt → kilowatt

**Unit Conversion**

1. Convert 50 metres to kilometres:

$$50\,\text{m} \times \frac{1\,\text{km}}{1000\,\text{m}} = 0.05\,\text{km}$$

Conversion factor: 1 km = 1000 m

2. Convert 50 kilometres to metres:

$$50\,\text{km} \times \frac{1000\,\text{m}}{1\,\text{km}} = 50{,}000\,\text{m}$$

Conversion factor: 1000 m = 1 km

## ACTIVITY

### 5.3 SI Prefixes

1. Define the term *prefix*.
2. Write the names and symbols of the SI prefixes that mean the following:

| | |
|---|---|
| (a) $\times 10$ | (g) $\times 0.1$ |
| (b) $\times 100$ | (h) $\times 0.01$ |
| (c) $\times 1000$ | (i) $\times 0.001$ |
| (d) $\times 10^{6}$ | (j) $\times 0.000001$ |
| (e) $\times 10^{3}$ | (k) $\times 10^{-3}$ |
| (f) $\times 10^{2}$ | (l) $\times 10^{-6}$ |

3. Write symbols for the following units:

| | |
|---|---|
| (a) kilogram | (e) megacandela |
| (b) centimetre | (f) microampere |
| (c) millisecond | (g) hectometre |
| (d) decimetre | (h) microgram |

4. One mg is equal to:

| | |
|---|---|
| (a) 1000 g | (c) 0.001 g |
| (b) 100 g | (d) 0.01 g |

5. Complete the following statements:

| | |
|---|---|
| (a) 1 cm = ___ m | (e) 1 dm = ___ m |
| (b) 1 μg = ___ g | (f) 1 hm = ___ m |
| (c) 1 ms = ___ s | (g) 1 Mcd = ___ cd |
| (d) 1 kA = ___ A | (h) 1 dam = ___ m |

6. Write the most suitable unit for measuring:
   (a) the height of a building
   (b) the distance from Suva to Labasa
   (c) the height of a book
   (d) the height of a line of text
   (e) the thickness of a page of a book
7. Write the number of:
   (a) centimetres in one metre
   (b) milligrams in one gram
   (c) microamperes in one ampere
   (d) decimetres in one metre
   (e) candelas in one kilocandela
8. Write in grams:

| | | |
|---|---|---|
| (a) 1 kg | (d) 2 kg | (g) 22 kg |
| (b) 1 mg | (e) 2 mg | (h) 15 mg |
| (c) 1 μg | (f) 5 μg | (i) 35 μg |

9. Write in metres:

| | | |
|---|---|---|
| (a) 1 km | (d) 3 km | (g) 25 km |
| (b) 1 cm | (e) 5 cm | (h) 60 cm |
| (c) 1 μm | (f) 4 μm | (i) 120 μm |

10. Write in seconds:

| | | |
|---|---|---|
| (a) 1 ms | (d) 0.1 ms | (g) 10 μs |
| (b) 10 ms | (e) 0.01 ms | (h) 100 μs |
| (c) 100 ms | (f) 1 μs | (i) 0.1 μs |

## SUSBSTRAND 2.2 CONVERSION OF UNITS

**LESSON 31**

LO: Convert from one unit to another.

### CONVERTING BETWEEN UNITS

**-Conversion factor:** is a fraction that has the units we are converting to in the numerator and the units we are converting from in the denominator.

- The value of a conversion factor is **1**, so it does not change the value of the measurement. It only changes the units.

**Example 1**

**Convert 25 cm to m**

Convert 25 cm to metres

$$25\ \text{cm} \times \frac{1\ \text{m}}{100\ \text{cm}} = 0.25\ \text{m}$$

Conversion factor (1 m = 100 cm)

Convert 0.25 m to centimetres

$$0.25\ \text{m} \times \frac{100\ \text{cm}}{1\ \text{m}} = 25\ \text{cm}$$

Conversion factor (100 cm = 1 m)

**Example 2** A school day lasts about 7 hours. How many seconds are there in a school day?

(i) Hours to minutes

$$7\ \text{h} \times \frac{60\ \text{min}}{1\ \text{h}} = 420\ \text{min}$$

There are 60 min in 1 h

(ii) Minutes to seconds

$$420\ \text{min} \times \frac{60\ \text{s}}{1\ \text{min}} = 25{,}200\ \text{s}$$

There are 60 seconds in 1 min

**Unit Conversion**

1. Convert 50 metres to kilometres:

$$50\ \text{m} \times \frac{1\ \text{km}}{1000\ \text{m}} = 0.05\ \text{km}$$

Conversion factor: 1 km = 1000 m

2. Convert 50 kilometres to metres:

$$50\ \text{km} \times \frac{1000\ \text{m}}{1\ \text{km}} = 50{,}000\ \text{m}$$

Conversion factor: 1000 m = 1 km

## ACTIVITY

### 5.4 Converting Between Units

1. Define the term *conversion factor*.
2. Complete the following statements:

| | | | |
|---|---|---|---|
| (a) | 1 min = ___ sec | (g) | 1 kg = ___ g |
| (b) | 1 h = ___ min | (h) | 1 g = ___ mg |
| (c) | 1 km = ___ m | (i) | 1 g = ___ µg |
| (d) | 1 m = ___ cm | (j) | 1 A = ___ mA |
| (e) | 1 m = ___ mm | (k) | 1 A = ___ µA |
| (f) | 1 m = ___ µm | (l) | 1 s = ___ ms |

3. A conversion factor has a value of 1 because the numerator and denominator are equal. Fill the missing values in the conversion factors below:

(a) $\frac{1\text{h}}{\_\_\_\text{min}}$ (d) $\frac{1\text{m}}{\_\_\_\text{cm}}$ (g) $\frac{1\text{g}}{\_\_\_\text{mg}}$

(b) $\frac{\_\_\_\text{sec}}{1\text{min}}$ (e) $\frac{\_\_\_\text{mm}}{1\text{m}}$ (h) $\frac{1\text{kg}}{\_\_\_\text{g}}$

(c) $\frac{1\text{km}}{\_\_\_\text{m}}$ (f) $\frac{1\text{m}}{\_\_\_\mu\text{m}}$ (i) $\frac{\_\_\_\mu\text{g}}{1\text{g}}$

4. Choose the conversion factors from question 3 that you would use to convert from:
   (a) minutes to seconds
   (b) metres to kilometres
   (c) grams to micrograms
   (d) centimetres to metres
5. Write your own conversion factors to change:
   (a) hours to minutes
   (b) seconds to minutes
   (c) millimetres to metres
   (d) kilograms to grams
6. Convert the following measurements to metres:

Example: $10\ \text{cm} \times \frac{1\text{ m}}{100\ \text{cm}} = 0.1\text{ m}$

| | | | | | |
|---|---|---|---|---|---|
| (a) | 5 cm | (d) | 114.5 cm | (g) | 119.3 mm |
| (b) | 3.5 cm | (e) | 5 mm | (h) | 25 µm |
| (c) | 89 cm | (f) | 77 mm | (i) | 1349 µm |

7. Convert the following measurements to the units indicated:

Example: $0.03\ \text{m} \times \frac{100\text{ cm}}{1\ \text{m}} = 3\text{ cm}$

| | | | |
|---|---|---|---|
| (a) | 0.05 m to cm | (d) | 2.45 m to cm |
| (b) | 0.007 m to mm | (e) | 1.135 m to mm |
| (c) | 0.0005 m to µm | (f) | 0.00084 m to µm |

8. Convert:
   (a) 24 seconds to minutes
   (b) 13.9 kilograms to grams
   (c) 2.43 centimetres to metres
   (d) 0.0093 grams to micrograms
9. A cyclist travelled 0.5 km in 90 seconds.

   (a) Convert 90 seconds to hours.
   (b) Find the cyclist's speed in km/h.
10. B5 paper (the size of this book) is 176 mm wide by 250 mm high.
    (a) How many millimetres are there in 1 centimetre?
    (b) Write a conversion factor to convert millimetres to centimetres.
    (c) Use your conversion factor to find the size of B5 paper in centimetres.
11. The cost of posting a book to the United States is $10 per kilogram. You need to post a book that weighs 475 g.
    (a) How many grams are there in 1 kg?
    (b) Write a conversion factor with kg in the numerator and g in the denominator.
    (c) Use your conversion factor to find the mass of the book in kg.
    (d) Find the cost of posting the book.
12. Television screens are measured in inches diagonally:


    (a) If 1" = 2.54 cm, write a conversion factor to change inches to centimetres.
    (b) Find the size of a 25" (25 inch) television in cm.
    (c) Find the size of a 14" television in cm.
    (d) If a television screen measures 48 cm, what is its size in inches?

## SUSBSTRAND 2.3 STANDARD FORM

### LESSON 32

LO: Write numbers in standard form and vice versa

### STANDARD FORM

In standard form, numbers are represented as products of powers (indices) of 10. Formally, standard form is $a \times 10^b$, where $1 \leq a < 10$ and $b$ is an integer.

- In **standard form**, numbers are represented as products of powers of 10.
- On your calculator "$1^{12}$" means $1 \times 10^{12}$.
- To enter $1 \times 10^{12}$ on your calculator, type 1 EXP 12.

### RULES OF STANDARD FORM

| Rule | Formally | Example |
|---|---|---|
| 1. Numbers are represented as products of powers of 10. | $a \times 10^b$ | $2 \times 10^3$<br>Not $2 \times 11^3$ |
| 2. $a$ is a decimal between 1 and 10 (or between -1 and -10) | $1 \leq a < 10$ | $2.5 \times 10^5$<br>Not $25 \times 10^4$ |
| 3. $b$ is an integer | $b \in I$ | $1.0 \times 10^{-2}$<br>Not $1.0 \times 10^{-2.5}$ |

### EXAMPLES

Convert **a**-d to standard form. **e-h** to ordinary decimals.

**(a)** $20 = 2 \times 10 = 2 \times 10^1$

**(b)** $300 = 3 \times 100 = 3 \times 10^2$

**(c)** $0.2 = 2 \times 0.1 = 2 \times 10^{-1}$

**(d)** $0.03 = 3 \times 0.01 = 3 \times 10^{-2}$

**(e)** $5 \times 10^2 = 5 \times 100 = 500$

**(f)** $1.5 \times 10^3 = 1.5 \times 1000 = 1500$

**(g)** $1 \times 10^{-1} = 1 \times 0.1 = 0.1$

**(h)** $2.5 \times 10^{-2} = 2.5 \times 0.01 = 0.025$

### ACTIVITY

Write each number in standard form:

| | | |
|---|---|---|
| (a) 100 | (d) 1 | (g) 0.001 |
| (b) 10 | (e) 0.01 | (h) 10,000 |
| (c) 0.1 | (f) 1000 | (i) 100,000 |

## SUSBSTRAND 2.4 ACCURACY OF MEASUREMENETS

**LESSON 33**

LO: Write measurements correct to nearest unit required.

## ROUNDING

- Is done so that the answer has the same number of significant figures as the number in the calculation with the least number of significant figures.

- An **accurate** measurement is as close to the true value as we can measure.
- An **approximate** measurement is rounded off for convenience.
- In general, the more decimal places a measurement has, the more accurate it is.

**EXAMPLES**

1. 16.23 ÷ 1.72 = 9.4360465 ( ROUNDED OFF TO 2 DP = 9.44)
2. $\frac{19.35\ X\ 4.2}{6.81}$ = 11.9 ( 1 DP)

***WHENEVER ROUNDING IS DONE, THE ACCURACY OF ROUNDING SHOULD BE STATED.**

**ACTIVITY**

Write to the nearest 10 m:

| | | |
|---|---|---|
| (a) 12 m | (c) -33.5 m | (e) 933.4 m |
| (b) 49 m | (d) 1687 m | (f) 299 m |

Write to the nearest 0.1 g:

| | | |
|---|---|---|
| (a) 0.35 g | (c) 13.26 g | (e) 1.1189 g |
| (b) 1.94 g | (d) 0.006 g | (f) 0.95 g |

Write the following measurements correct to the nearest whole number (i.e. round them to the nearest whole number):

| | | |
|---|---|---|
| (a) 3.4 m | (c) 12.5 g | (e) 1.05 A |
| (b) 5.6 cm | (d) 9.49 mL | (f) 10.92 m |

**LESSON 34**

LO: Give appropriate upper and lower bounds for data given to a specified accuracy.

: Write an inequality for the actual measure.

## ACCURACY OF A REPORTED MEASUREMENT

- The **reported value** (recorded value) of a measurement is the value that is given.
- The **actual value** of a measurement is from ½ unit below to ½ unit above the reported value.

### EXAMPLES

1. The length of a pen is recorded as being 14.4cm, its actual length lies between 14.35 and 14.45cm.
2. The time taken for a car to travel down a street was written down as 12 s. The actual time taken lies between 11.5 s and 12.5 s.

### STEPS

To calculate the limits of accuracy for any measurement, proceed as follows:

Lower limit: reduce the last significant figure by 1 and 'add a 5' to the end of the number (adding a decimal point if required). Thus 14.4 becomes 14.35, and 12 becomes 11.5. Note that 2.50 becomes 2.495.

Upper limit: 'add a 5' to the end of the number (adding a decimal point if required). Thus 14.4 becomes 14.45, and 12 becomes 12.5.

**Example F:** The side length of a square is given as 11.5 cm. Between what limits does its perimeter lie?

**Solution:** The lower limit of accuracy of the measurement is 11.45 cm. The upper limit of accuracy of the measurement is 11.55 cm. Therefore the perimeter lies between $4 \times 11.45 = 45.8$ cm and $4 \times 11.55 = 46.2$ cm.

Suppose you are told that a pencil is 8 cm long. What is the actual (true) length of the pencil?

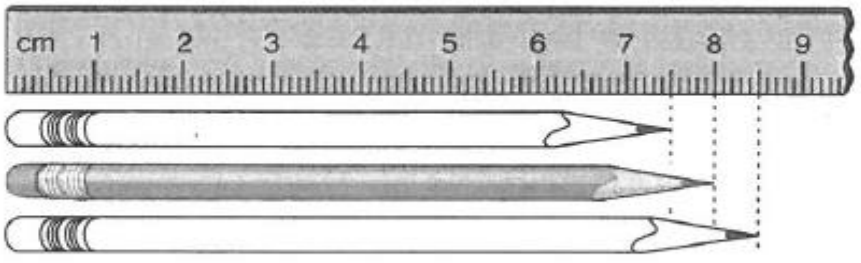


**Figure 5.13** If the length of a pencil is *reported to be* 8 cm, its length was measured to the nearest cm. Thus, the pencil could have been as short as 7.5 cm or as long as 8.5 cm.

**Expressing a measurement as an inequality**

In figure 5.13, the *reported length* of the pencil is 8 cm:

$$\ell = 8 \text{ cm}$$

The *actual length* is between 7.5 and 8.5 cm. We can write this as an inequality:

$$7.5 \leq \ell < 8.5^*$$

*We say $< 8.5$ because if $\ell$ were equal to 8.5, it would have been rounded up to 9.

**ACTIVITY**

## 5.9 Reported Measurement

1. The length of a boat is reported as 12 m.
   (a) Its length is reported to the nearest ____.
   (b) What is half of this unit?
   (c) What is the largest possible length of the sailboat?
   (d) What is the smallest possible length?
   (e) Write the length as an inequality.


2. A lorry's mass is reported as 8.7 tonnes.
   (a) Find the largest and smallest possible masses of the lorry.
   (b) Express its mass as an inequality.

3. For each of the measurements below, write an inequality for the actual value:
   (a) 5 m (c) 22 L (e) 315 mL
   (b) 0.5 m (d) 1.7 mg (f) 0.22 A

4. The voltage in a power line is reported as 220 volts, to the nearest 10 volts.
   (a) Find the maximum and minimum possible voltages.
   (b) Write the voltage as an inequality.

5. Merilyn measured a line segment and announced that it was 10 cm long, to the nearest cm.
   (a) Draw three lines of different length that are "10 cm long to the nearest cm."
   (b) What is the length of the shortest line Merilyn could have measured?
   (c) What is the length of the longest line she could have measured?

6. The lengths of four pencils, 18, 12, 9, and 14 cm were measured to the nearest cm.
   (a) If the pencils were lined up end-to-end, what would be the greatest possible length?
   (b) What would be the smallest length?
   (c) Write the length as an inequality.